**Potentials for random substances**

**Prologue**

Now we want to investigate how we may determine the functional form of these potentials introduced previously. We will consider for simplicity that our substance is homogeneous and obeys the scaling relations introduced in the previous lecture. Then our potentials satisfy,



It will suffice, then, to determine the heat capacity CV(T,V) and equation of state p(T,V), to construct the potentials. For instance, forming the differential of U(T,V) we have:



What are these derivatives? Well,



and,



So we have:



A similar procedure for the entropy would yield:



What are these derivatives? Well,



and,



So we have:



With U(T,V) and S(T,V), we can then invert to get S in terms of its canonical variables, as well as the other potentials as desired.

**Ideal molecular gas**

Suppose that the gas particles are molecular, as is allowed even above, and that we take into account the bond strength of the molecule, φ (a negative #) so that the internal energy is U = KE + Nφ. What changes then? Well I would suppose that the entropy is function of KE, V, and N, really, so that we could say:



What then is T?



What about pressure?



and so that is as before as well, which is to be expected. And so the entropy would be the same function of temperature as before,



and the free energy would therefore be:



and Gibbs energy is:



**Air balloon**

Let’s consider the entropy for an air balloon. I think it should be a gas basically, surrounded by some sort of membrane under tension. I’ll presume the energy of the membrane is something like PE = γA. And so we can posit the following entropy formula:



since the entropy would just depend on the kinetic energy of the gas molecules inside? Now assuming a spherical balloon, A and V are not independent variables of course. But V = (4/3)πR3, and A = 4πR2. But if I just subsume the constants into γ, along with the two, then I can say that:



So what would be the pressure formula for such an air bubble? Well, p = -∂U/∂V)S and this would be, taking the implicit derivative:



And,



So can say,



Could put this in terms of just V of course. First term is the usual, and the second is the contribution from surface tension. What is this in terms of T? Well, T = ∂U/∂S so…



so we have the expected expression for U. Solving for T and plugging into the pressure formula we get:



which is also expected I suppose. The pressure comes from the kinetic and potential contributions. I think the pressure is called the Young-Laplace equation:

